Reprinted from IEEE TRANSACTIONS
ON *ELECTRON DEVICES*
Volume ED-16, Number 4, April, 1969
pp. 401-402

Effect of Failure Kinetics on Time-to-Failure Distributions

ROBERT G. STEWART, SENIOR MEMBER, IEEE

***Abstract*—The kinetics or time dependence of a device quantity has an explicit effect on the time-to-failure distribution resulting from a given distribution of device degradation coefficients. In particular, for *n*th power kinetics, the larger the value of *n*, the smaller the deviation from the mean.**

Introduction

In a previous paper [1] we examined the interaction between the time dependence and the stress dependence of failure mechanisms that causes the activation energy of the failure rate to differ from the activation energy of the failure mechanism. The kinetics of the device quantity was also shown to be of crucial importance in determining how the failure rate depends on the failure limit. In this paper we shall examine another interesting effect, how the time dependence of the failure process can map the form of the distribution of the entities which give rise to the distribution into a different distribution of times-to-failure.

Review of Terms

Suppose that there is some particular aspect of a device which we are concerned with; let us denote this as the *device quantity* q_i. The device quantity can depend on material constants, structural dimensions, etc., which can be denoted as *properties* p_j. The device quantity can vary in time due to stresses s_k, due to signal changes, and due to the effect of failure mechanisms acting through the properties,

$$q_i = q_i(p_j, s_k, t).$$

The time-to-failure for a device τ_i determines the failure rate of a device F_i through the definition

$$F_i = \frac{1}{\tau_i} \cdot \tag{1}$$

The time-to-failure is simply the time required for the device quantity to change by an amount equal to the failure limit δq_F. This statement or condition can be restated by the following equation:

$$\delta q_F = \int_0^{\tau_i} \frac{dq_i}{dt}\, dt = \int_0^{\tau_i} \left[\sum_j \frac{\partial q_i}{\partial p_j} \frac{\delta p_j}{\delta t} + \frac{\partial q_i}{\partial t} \right] dt$$

where we have presumed the applied stresses are constant; if there exists any *explicit* stress dependence of the device quantity, a stress term

$$\sum_k \frac{\partial q_i}{\partial s_k} \frac{\delta s_k}{\delta t}$$

must be added. The derivative terms, e.g., $\delta p_j/\delta t$, denote partial derivatives if the other variables of the property are presumed held constant, otherwise they are total derivatives; an excellent discussion of the cases where one approach or the other is advantageous is given by Sokolnikoff [2].

If we neglect the explicit time dependence of the device quantity $\partial q_i/\partial t$, then the above equation, which we shall call the constraint equation, becomes

$$\delta q_F = \int_0^{\tau_i} \sum_j \frac{\partial q_i}{\partial p_j} \frac{\delta p_j}{\delta t}\, dt. \tag{2}$$

This equation can be considered as one of the basic equations of reliability. The individual terms in the sum are interesting in themselves. Let us denote the kinetic sensitivity as

$$\phi_j \equiv \frac{\partial q_i}{\partial p_j} \frac{\delta p_j}{\delta t} \tag{3}$$

where the first partial derivative represents the sensitivity aspect, and $\delta p_j/\delta t$ denotes the kinetics of change of a particular property due to the action of the failure mechanism. By denoting the failure function as the sum of all the kinetic sensitivities

$$\psi \equiv \sum_j \phi_j, \tag{4}$$

the constraint equation simplifies to

$$\delta q_F = \int_0^{\tau_i} \psi dt.$$

Hence, the failure function represents the time rate of change of the device quantity due to the failure mechanisms.

Mapping of Variability

Now let us consider a distribution made up of a number of devices. For a device located at the mean of the distribution, the constraint equation is

$$\delta q_F = \int_0^{\bar{\tau}} \bar{\psi} dt \tag{5}$$

where $\bar{\tau}$ and $\bar{\psi}$ are the mean time-to-failure and the mean of the failure function. The constraint equation holds true, of course, for any other device in the distribution

Manuscript received October 17, 1968; revised December 18, 1968. This paper was presented at the IEEE Reliability Physics Symposium, Washington, D. C., December 1968.
The author is with Lockheed Palo Alto Research Laboratory, Palo Alto, Calif. 94304.

$$\delta q_F = \int_0^{\tau_i} \psi_i dt.$$

Suppose we regard the τ_i and ψ_i for the ith device as variations from the mean, or

$$\tau_i = \bar{\tau} + \delta\tau_i, \quad \text{and} \quad \psi_i = \bar{\psi} + \delta\psi_i.$$

Then the constraint equation for that device becomes

$$\delta q_F = \int_0^{\bar{\tau}} \bar{\psi} dt + \int_0^{\bar{\tau}} \delta\psi_i dt + \int_{\bar{\tau}}^{\tau_i} \bar{\psi} dt + \int_{\bar{\tau}}^{\tau_i} \delta\psi_i dt. \quad (6)$$

But by (5) the first term on the right-hand side is equal to the left-hand side, which means that the last three terms on the right are constrained to sum to zero! Hence,

$$\int_0^{\bar{\tau}} \delta\psi_i dt = \int_{\tau_i}^{\bar{\tau}} (\bar{\psi} + \delta\psi_i) dt. \quad (7)$$

However, the mean value theorem for integrals says that we can find a mean value of the integrand such that

$$\overline{(\bar{\psi} + \delta\psi_i)}[\bar{\tau} - \tau_i] = \int_{\tau_i}^{\bar{\tau}} (\bar{\psi} + \delta\psi_i) dt. \quad (8)$$

Then, neglecting the $\delta\psi_i$ term in the left-hand side of (8) and substituting into (7), we have

$$\bar{\tau} - \tau_i \simeq \frac{\int_0^{\bar{\tau}} \delta\psi_i dt}{\bar{\psi}}, \quad (9)$$

which is an approximation valid for narrow distributions.

To examine the effects of the kinetics, suppose that the device quantity varies in time as

$$q_i = a_i p = a_i t^n$$

and that it is the variations of the constants a_i, which we shall call the degradation coefficient, that give rise to the distributions of times-to-failure. Since only one property is present, the failure function is just the kinetic sensitivity, so

$$\psi_i = \phi = \frac{\partial q}{\partial p} \frac{\delta p}{\delta t}$$
$$= a_i n t^{n-1}.$$

Now the mean value of the failure function is

$$\bar{\psi} = \bar{a} n (\bar{\tau})^{n-1}$$

and

$$\delta\psi_i = \frac{\partial \psi_i}{\partial a_i} \delta a_i = n t^{n-1} \delta a_i \quad (10)$$

where the deviation of the degradation coefficient of the ith device from the mean is

$$\delta a_i \equiv a_i - \bar{a}.$$

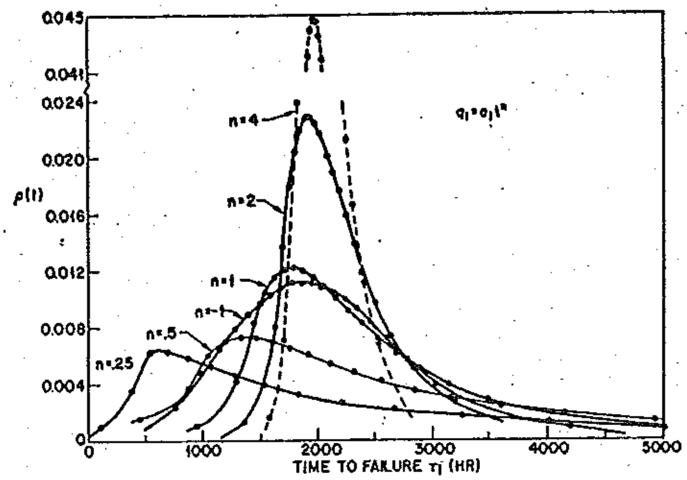

Fig. 1. Effect of failure mechanism kinetics on mapping of device variability into time-to-failure distributions.

Then substituting (10) in (9), and performing the time integration yields

$$\bar{\tau} - \tau_i \simeq \frac{\bar{\tau}\delta a_i}{n\bar{a}}$$

or

$$\tau_i \simeq \left[1 - \frac{\delta a_i}{n\bar{a}}\right]\bar{\tau}. \quad (11)$$

The appearance of the power of the time dependence of the device quantity n in the denominator of (11) thus has proven the following general conclusion, which we shall denote as Theorem 5 (Theorems 1–4 can be found in [1]).

Theorem 5

The time dependence of the device quantity has an explicit effect on the mapping of a distribution of degradation coefficients into a distribution of times-to-failure; in particular, for nth power kinetics, the larger the value of n, the smaller the deviation from the mean.

This result suggests that failure processes with catastrophic type of kinetics, $n \gg 1$, will give rise to narrower, more peaked failure distributions than saturating kinetics, $n \ll 1$. For negative values of n, the distribution of times-to-failure is reflected about the mean. Fig. 1 portrays this mapping of a distribution of degradation coefficients corresponding to a Weibull distribution with shape parameter $\beta = 3.0$ into distributions of times-to-failure for different values of the power of the kinetics. The validity of Theorem 5 is clearly evident, demonstrating again the importance of ascertaining and knowing the time dependence as well as the stress dependence of failure mechanisms.

References

[1] R. G. Stewart, "A causal redefinition of failure rate—theorems, stress dependence, and application to devices and distributions," *IEEE Trans. Reliability*, vol. R-15, pp. 95–114, December 1966.

[2] I. S. and E. S. Sokolnikoff, *Higher Mathematics for Engineers and Physicists*, 1st ed. New York: McGraw-Hill, 1934, ch. 5, p. 107.