ECON 159a Solution Set 1

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November 15, 2009

1 Solution 1

player	i, j
strategy of a particular player i	s_i
set of strategies for player i	S_i
payout (utile)	$u_i(s_1, ...s_N)$
strategy choices for everyone else but player i	s_{-i}

1.1 (a)

A strictly dominated strategy A (s'_i) means that, regardless of the opponent's strategy, there is a higher payoff for (strictly dominating) strategy B (s_i). Formally player i's strategy s'_i is strictly dominated by player i's strategy s_i if $u_i(s_i, s_{-i}) > u_i(s'_i, s_{-i})$ for all s_{-i}.

1.2 (b)

A weakly dominated strategy A (s'_i) means that, regardless of the opponent's strategy, there is at least as good of a payoff for (weakly dominating) strategy B (s_i), and, for at least one of the opponent's strategies, there is a higher payoff for strategy B. Formally player i's strategy s'_i is weakly dominated by player i's strategy s_i if both $u_i(s_i, s_{-i}) >= u_i(s'_i, s_{-i})$ for all s_{-i} and also $u_i(s_i, s_{-i}) > u_i(s'_i, s_{-i})$ for at least one s_{-i}.

1.3 (c)

T is strictly dominated by M for player 1; C is weakly (but not strictly) domainted by L for player 2.

		player 2		
		L	C	R
	T	1,1	-1,0	0, -1
player 1	M	2,0	0,0	1, 0
	B	1,0	1,0	0, 0

2 Solution 2

2.1 (a)

There are no strictly dominated strategies. M is weakly dominated by D for player 1 and c is weakly dominated by r for player 2.

2.2 (b)

If player 2 assumes player 1 will never play weakly dominated M, and player 1 assumes player 2 will never play weakly dominated c, these strategies can be deleted. After deleting M and c, D is weakly dominated by T for player 1 and r is weakly dominated by l for player 2. A second round of deletion would leave just T and l.

2.3 (c)

In the first round of deletion, the worst-case utile was 1 for both players. In the second round of deletion, again, the worst-case util was 1 for both players. Iteratively deleting the weakly dominated strategies left nothing but the worst-case util of 1 for both players.

3 Solution 3

3.1 (a)

The payoff matrix is listed in table 3.a. A graph of u_i is included in Figure 1. $s_i(1)$ is strictly dominated by $s_i(2, 3..7)$. Because of symmetry, $s_i(10)$ is certainly not any better than $s_i(1)$. $s_i(8)$ and $s_i(9)$ are worse than $s_i(1)$ when the opponent picks an adjacent or nearly adjacent position.

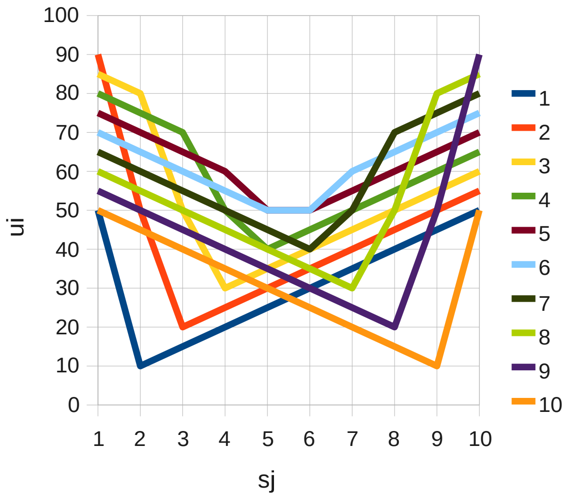
Table 1: 3.a

	1	2	3	4	5	6	7	8	9	10
1	50,50	10,90	15,85	20,80	25,75	30,70	35,65	40,60	45,55	50,50
2	90,10	50,50	20,80	25,75	30,70	35,65	40,60	45,55	50,50	55,45
3	85,15	80,20	50,50	30,70	35,65	40,60	45,55	50,50	55,45	60,40
4	80,20	75,25	70,30	50,50	40,60	45,55	50,50	55,45	60,40	65,35
5	75,25	70,30	65,35	60,40	50,50	50,50	55,45	60,40	65,35	70,30
6	70,30	65,35	60,40	55,45	50,50	50,50	60,40	65,35	70,30	75,25
7	65,35	60,40	55,45	50,50	45,55	40,60	50,50	70,30	75,25	80,20
8	60,40	55,45	50,50	45,55	40,60	35,65	30,70	50,50	80,20	85,15
9	55,45	50,50	45,55	40,60	35,65	30,70	25,75	20,80	50,50	90,10
10	50,50	45,55	40,60	35,65	30,70	25,75	20,80	15,85	10,90	50,50

3.2 (b)

Strategy 1 is weakly dominated by strategy 2. The worst u_i for $s_i(1)$ is to receive only 5 votes (other players choose $s(1)$ and $s(2)$), but otherwise u_i for $s_i(1) >= 10$ as player 1's vote is not split. The worst u_i for

Figure 1: Player 1 utile



$s_i(2)$ is to receive 10 votes when another player chooses $s(3)$ and the third player chooses $s(1)$ or $s(2)$, but otherwise u_i for $s_i(2) > 10$. $s_i(2) > s_i(1)$ because $s_i(2)$ includes at least a portion of $s(1)$'s 10 votes.

$$\begin{aligned} s_i(1, s_{-i}) &< s_i(2, s_{-i}), \text{ except} \\ s_i(1,3,2) &= s_i(2,3,2) \\ s_i(1,2,3) &= s_i(2,2,3) \\ &= 10 \end{aligned}$$

Strategy 1 is also weakly dominated by strategy 3, for the same reason. The worst u_i for $s_i(3)$ is to receive 10 votes ($s_j(2)$, $s_k(4)$), but otherwise $s_i(3)$ gives the opportunity of more votes.

$$\begin{aligned} s_i(1, s_{-i}) &< s_i(3, s_{-i}), \text{ except} \\ s_i(1,4,2) &= s_i(3,4,2) = 10 \\ s_i(1,4,3) &= s_i(3,4,3) = 15 \\ s_i(1,2,4) &= s_i(3,2,4) = 10 \\ s_i(1,3,4) &= s_i(3,3,4) = 15 \end{aligned}$$

After iteratively deleting strategies 1 and 10, strategy 2 is not dominated by any other pure strategy s_i in the reduced game. There are always cases where $u_i s_i(2)$ does better than s'_i when s_j and s_k are adjacent to s'_i because in the $s_i(2)$ case, s_{-i} can't be 1 by definition.

$$\begin{aligned} s_i(3,2,4) &< s_i(2,2,4) \\ s_i(4,3,5) &< s_i(2,3,5) \\ s_i(5,3,6) &< s_i(2,3,6) \\ s_i(6,4,7) &< s_i(2,4,7) \\ s_i(7,5,7) &< s_i(2,5,7) \\ s_i(8,4,8) &< s_i(2,4,8) \\ s_i(9,2,8) &< s_i(2,2,8) \end{aligned}$$

4 Solution 4

4.1 (a)

(See Farquharson, The theory of voting, or McKelvey/Niemi)

$s_1(a)$ strictly dominates $s_2(b)$ and $s_2(c)$. $s_1(a)$ wins a 7/9 of the time and in the other 2 cases, s_1 is irreveant. Assuming $s_1(a)$, the payoff for player 2 and 3 is:

	Member 3		
	a	b	c
a	1,0	1,0	1,0
Member 2 b	1,0	0,2	1,0
c	1,0	1,0	2,1

For s_2, $s_2(c)$ weakly dominates $s_2(b)$ and $s_2(a)$. For s_3, $s_3(a)$ is weakly dominated by $s_3(b)$ and $s_3(c)$.

4.2 (b)

After removing $s_2(a)$, $s_2(b)$, and $s_3(a)$, the predicted vote is $s(a, c, c)$. c will win, which is worst for 1, even though 1 had tiebreaker power.