ECON 159a Solution Set 2

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December 9, 2009

1 Penalty Shots Revisited

1.1 (a)

Neither player has a pure dominant strategy. For s_1, M does better than L for l, yet L does better than M for r; R does better than M for l, yet M does better than R for r; R does better than L for l, yet L does better than R for r. A similar argument can be made for s_2.

1.2 (b)

$s_2(m)$ is a best response (BR) for $s_1(M)$. However, $s_1(M)$ is not a BR for any belief about $s2$ (l's BR is R; m's better responses are L or R; r's BR is L).

1.3 (c)

No. Since $s_1(M)$ is not a BR to $s_2(m)$, s_1 will choose L or R. However, m is not a BR for L or R, thus s_2 should never play m.

1.4 (d)

A Nash equilibrium exists when there is a set of strategies with the property that no player benefits by changing strategies while other players keep their strategies unchanged. There is no such set:

Strategy	s_1 better response	s_2 better response
$s(L,l)$	R	-
$s(L,m)$	-	l
$s(L,r)$	-	l
$s(M,l)$	R	m
$s(M,m)$	L or R	-
$s(M,r)$	L	m
$s(R,l)$	-	r
$s(R,m)$	-	r
$s(R,r)$	L	-

2 Partnerships Revisited

2.1 (a)

We repeat the process of differentiating u_i, given the partners do the same amount of work, s, but first summing both u_1 and u_2 to determine the total revenue:

$$\begin{aligned} u_{1+2}(s_1, s_2) &= [4(s_1 + s_2 + bs_1 s_2)] - s_1^2 - s_2^2 \\ f(s) &= 4(s + s + bs^2) - s^2 - s^2 \\ f'(s) &= 8 + 8bs - 4s \\ s^{**} &= \frac{2}{1-2b} \end{aligned}$$

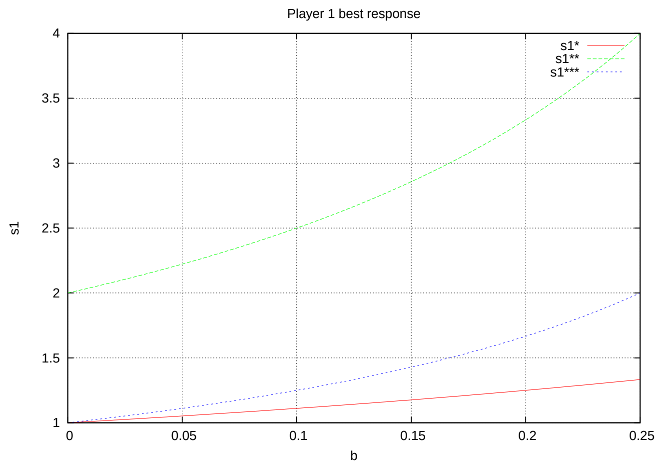
s^{**} is between 2 to 3 times the rationalizable effort $\frac{1}{1-b}$ (with $0 \leq b \leq 1/4$).

The midpoint of the effort range $s_i(2)$ provides the maximum shareable profit and minimum effort cost. Examining the extremes of s_1 and s_2 shows that $s_i(4)$ is no better than $s_i(0)$.

s_1	s_2	b	u_1	u_2
0	0	any	0	0
4	0	any	-8	8
0	4	any	8	-8
4	4	0	0	0
4	4	1/4	8	8

2.2 (b)

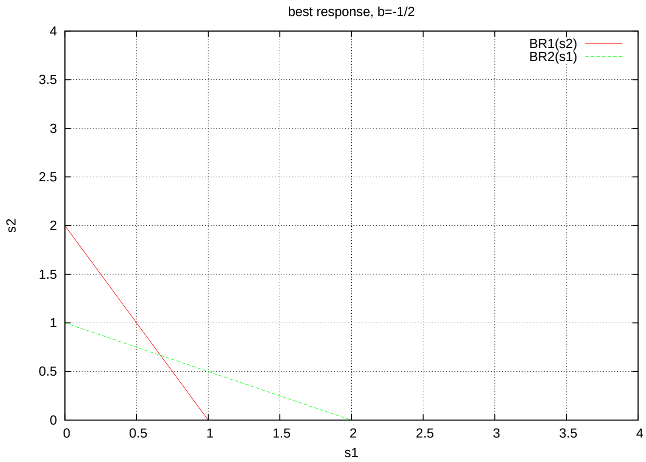
$$\begin{aligned} s_2 &= \frac{2}{1-2b} \\ BR_1(s_2) &= 1 + bs_2 \\ &= 1 + b\frac{2}{1-2b} \\ s_1^{***} &= \frac{1}{1-2b} \end{aligned}$$



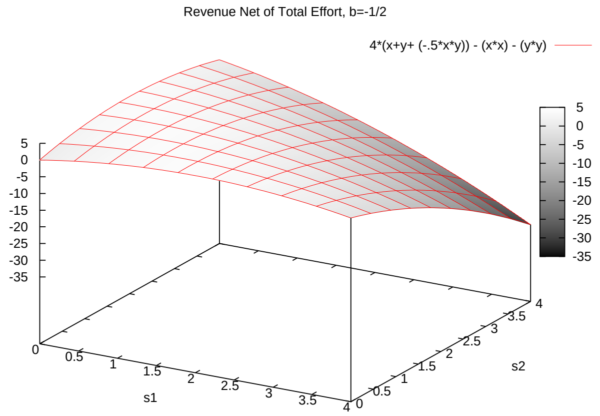
Player 1 will work half as diligently as player 2, if player 1 knows $s_2 = s^{**}$. The effort cost outweighs the benefit of working more.

2.3 (c)

$$\begin{aligned} s_1 &= 1 - \frac{s_2}{2} &= BR_1(s_2) \\ s_2 &= 1 - \frac{s_1}{2} &= BR_2(s_1) \end{aligned}$$



$$s_1^* = s_2^* = \frac{2}{3}$$



If the players contracted to provide the same amount of work, they would both work

$$\begin{aligned} s^{**} &= \frac{2}{1-2b}, b = -\frac{1}{2} \\ s^{**} &= 1 \end{aligned}$$

b	Nash equilibrium (s^*)	"Equal contract" (s^{**})	$u_i(s^*)$	$u_i(s^{**})$
$\frac{1}{4}$	$\frac{4}{3}$	4	$\frac{40}{9}$	8
$-\frac{1}{2}$	$\frac{2}{3}$	1	$\frac{16}{9}$	2

3 Nash Equilibria and Iterative Deletion

3.1 (a)

B and C are strictly dominated, leaving T, B, L, and R.

3.2 (b)

NE are (M,L) and (T,R).

3.3 (c)

Strictly dominated strategies are never a best response to anything; by definition, Nash equilibrium are a best response to something, so iteratively deleting strictly dominated strategies will never eliminate a Nash equilibrium.

4 Splitting the Dollar

4.1 (a)

NE(s_1, s_2), where $s_1 + s_2 = \$10$, as well as NE($10,$10).

4.2 (b)

NE($5.00,$5.00), NE($5.00,$5.01), NE($5.01,$5.00), NE($5.01,$5.01).

4.3 (c)

For (a) The NE involving strategies without whole-dollar units are eliminated.

For (b) The equilibria become NE($5,$5), NE($5,$6), NE($6,$5), NE($6,$6).